Franco-British (Complexified) Half-Space Matching

Simon Chandler-Wilde

Department of Mathematics and Statistics
University of Reading, UK

Relais 4 $\times$ 60:

É. Bécache, A.-S. Bonnet-Ben Dhia, C. Hazard, É. Lunéville

Ensta Paris, April 2014

Where is Reading?

Where is Reading?

University of Reading

It's near London

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It's pretty famous for work internationally on weather and climate

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E.g., today's newspaper headline ...

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E.g., today's newspaper headline ...

Reading University denies causing flooding in Dubai

telegraph.co.uk

This talk introduces the main ideas from ...

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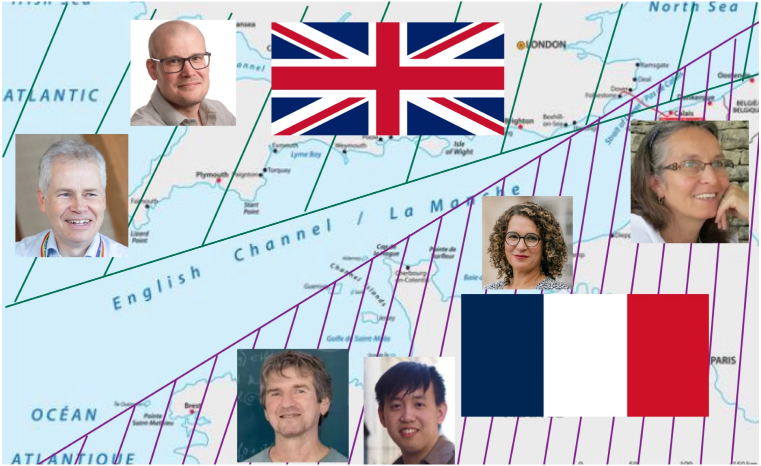

“**The complex-scaled half-space matching method**”, A.-S. Bonnet-Ben Dhia, S. N. Chandler-Wilde, S. Fliss, C. Hazard, K.-M. Perfekt, & Y. Tjandrawidjaja, *SIAM J. Math. Anal.* (2022)

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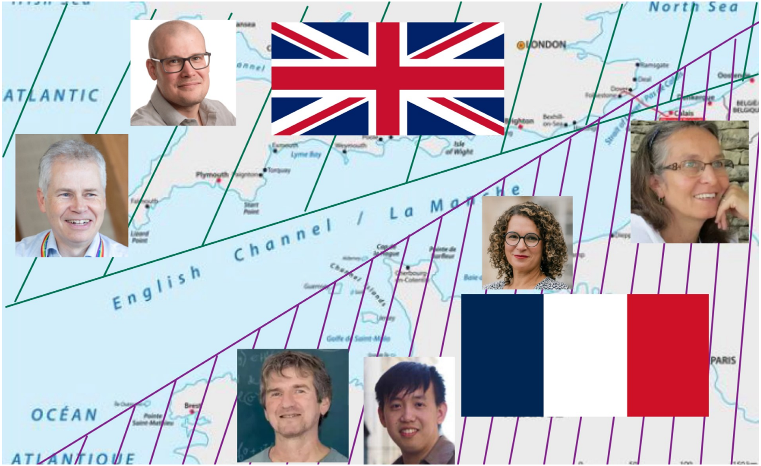

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as applied (in current work with **Anne-Sophie** and **Sonia**) to **diffraction by a right-angled wedge**.

BUT FIRST, some **Relais 4 $\times$ 60 memories** ...

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These will be in **English** or, in the recent words of French linguist **Bernard Cerquiglini**, in **French, badly spoken**

Starting with a **Relay Race par excellence**, the **Waves conference series** ...

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Waves 13: Minneapolis, USA in 2017: I finally **buy a phone with a camera**

.

Waves 14 Vienna, Austria in 2019: I have to miss this one as Dean but **follow the scores remotely** ...

Waves comes home to Paris in 2022: **we're finally together after Covid!**

And, of course, I have memories of **many joint research meetings** ...

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in person ...

ENSTA, 8 January 2019 (Sonia's Habilitation Defense)

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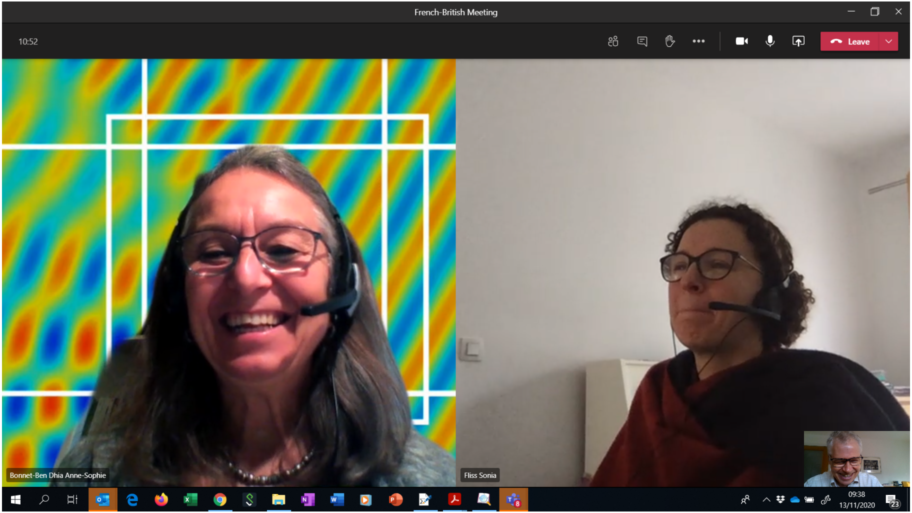
in person ...

Oberwolfach, September 2022

And on Zoom 13/11/20 ...



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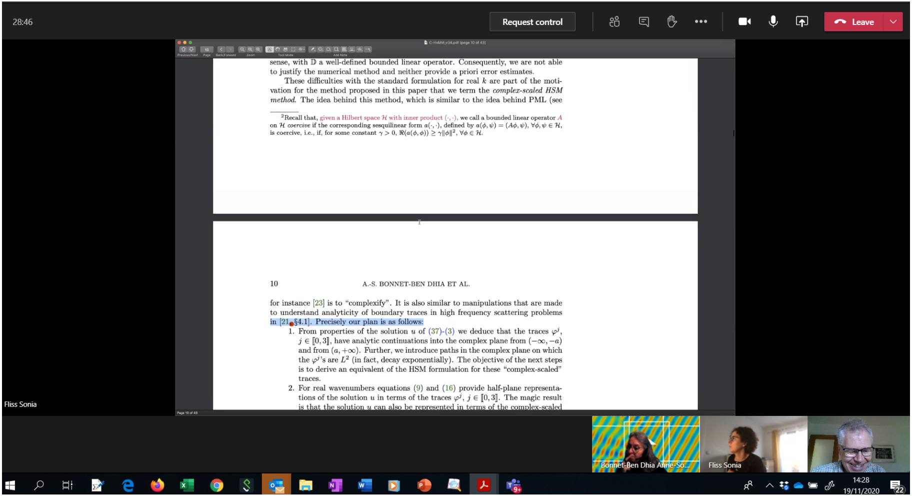
And a week later ...



But **enough of that** ... back to the

Franco-British (complexified) Half-Space Matching Method (HSMM)

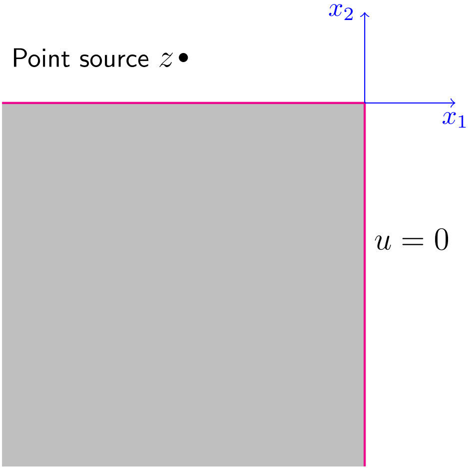
and ongoing work with **Anne-Sophie** and **Sonia** on

Diffraction by Right-Angled Wedges ...

Diffraction by a (right-angled) wedge – the HSMM way

u satisfies S.R.C. at ∞

$$\Delta u + k^2 u = \delta_z, \quad k > 0$$



The Half-Space Matching Method Philosophy

1. It is easy to solve explicitly Dirichlet problems in half-planes.
2. So express your solution in each of a number of overlapping half-planes using this explicit solution.
3. The HSMM equations are obtained by **enforcing compatibility** between these different half-plane representations.

Bonnet-BenDhia, Fliss, Tonnoir, *J. Comp. Appl. Math.* 2018

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$$u = g \quad \text{on} \quad \Sigma$$

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$$u = g \quad \text{on} \quad \Sigma$$

Solution is

$$u(x) = 2 \int_\Sigma \frac{\partial \Phi(x,y)}{\partial y_2} g(y) \, \mathrm{d}s(y), \quad x \in \Omega,$$

where

$$\Phi(x,y) := \frac{\mathrm{i}}{4} H_0^{(1)}(k|x-y|).$$

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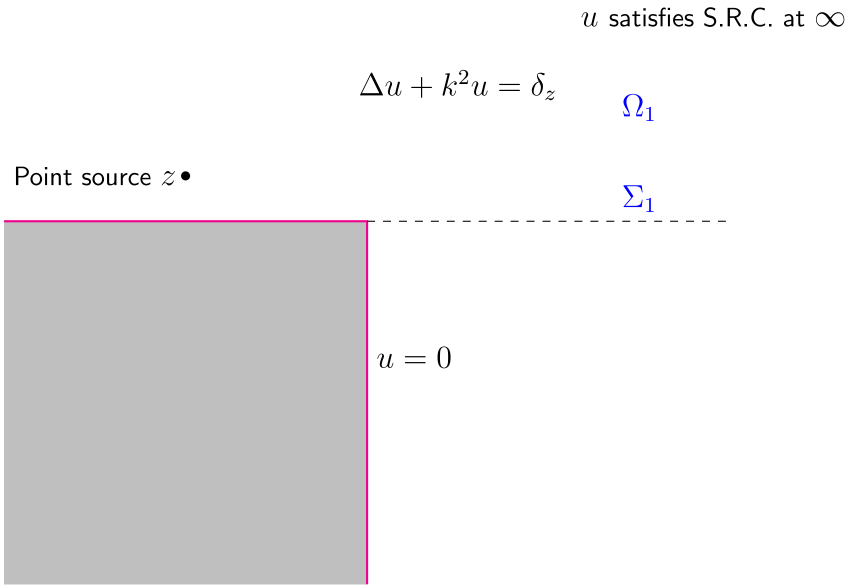
$$u(x) = G(x,z) + 2\int_{\Sigma} \frac{\partial \Phi(x,y)}{\partial y_2} g(y)\, \mathrm{d}s(y), \quad x \in \Omega,$$

where

$$G(x,z) := \Phi(x,z) - \Phi(x,z'), \quad \Phi(x,y) := \frac{\mathrm{i}}{4} H_0^{(1)}(k|x-y|).$$

Diffraction by a (right-angled) wedge – the HSMM way



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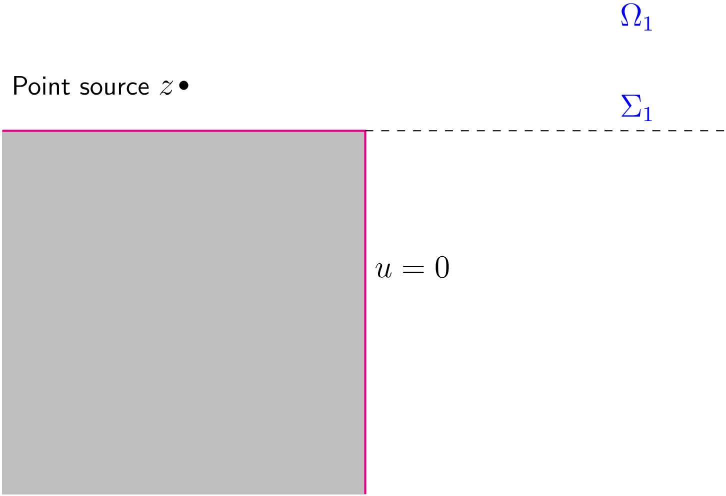

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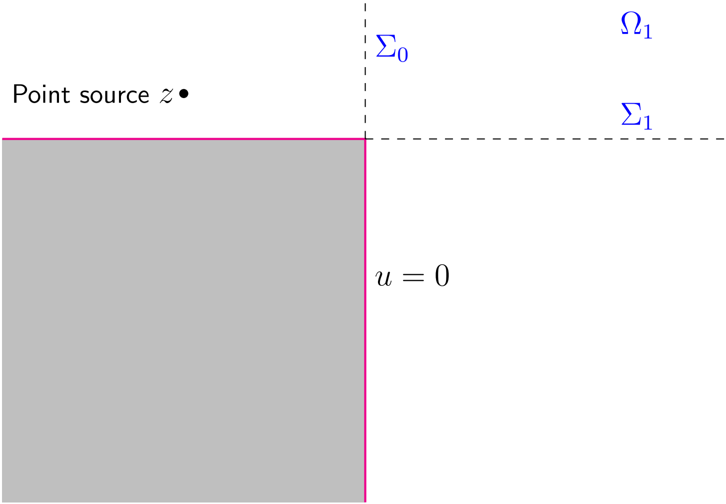

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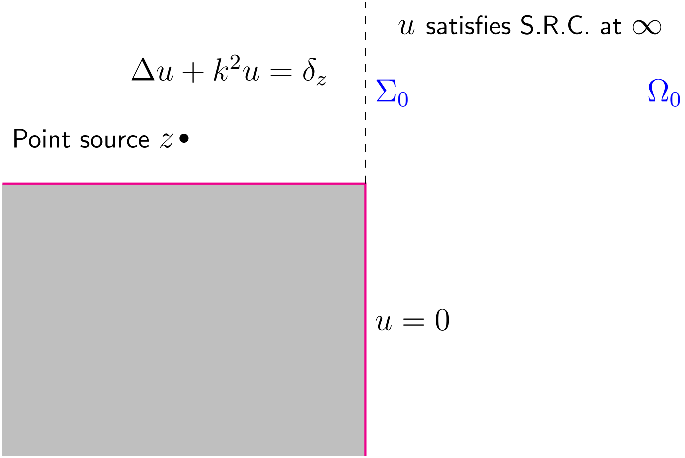

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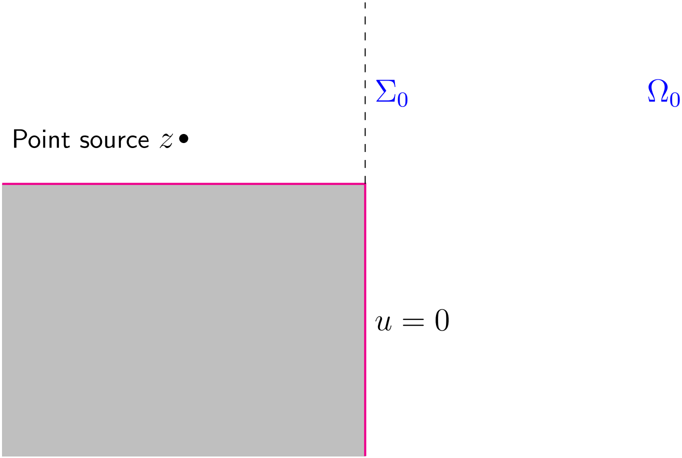

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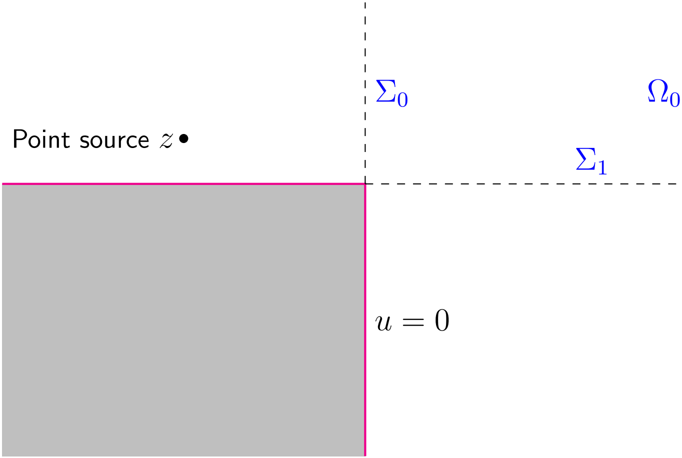

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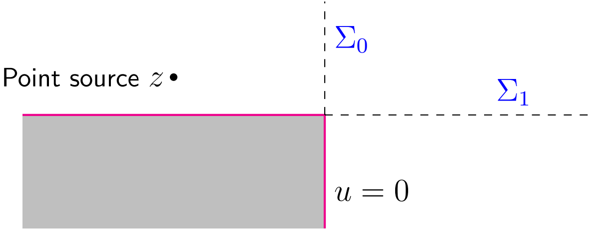
The HSMM integral equations



Two integral equations for unknowns $u|_{\Sigma_0}$ and $u|_{\Sigma_1}$:

$$
\begin{aligned}
u(x) &= G(x,z) + 2\int_{\Sigma_1} \frac{\partial \Phi(x,y)}{\partial y_2} u(y)\, \mathrm{d}s(y), \quad x \in \Sigma_0, \\
u(x) &= 2\int_{\Sigma_0} \frac{\partial \Phi(x,y)}{\partial y_1} u(y)\, \mathrm{d}s(y), \quad x \in \Sigma_1.
\end{aligned}
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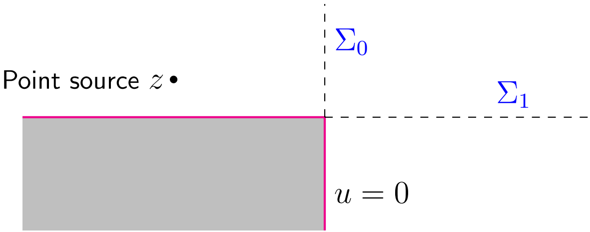

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These equations have exactly one solution (Bonnet-BenDhia, C-W, Fliss, *SIAM J. Appl. Math.* 2022) if one requires, additionally, that

$$u(x) = a_m \mathrm{e}^{\mathrm{i}kr} r^{-1/2} + O(r^{-3/2}), \quad \text{as } r := |x| \to \infty \text{ with } x \in \Sigma_m, \quad m = 0, 1.$$

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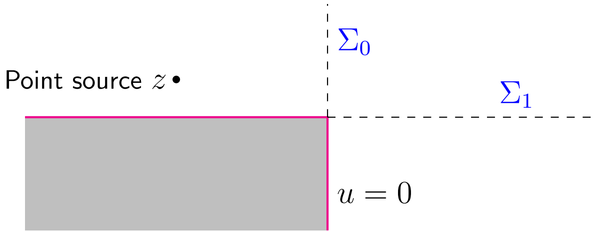

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Let $\varphi_0(s) := u((0,s))$ and $\varphi_1(s) := u((s,0))$, for $s \geq 0$.

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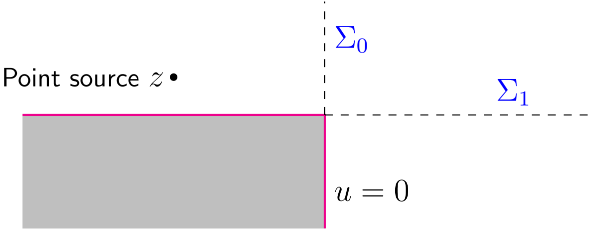

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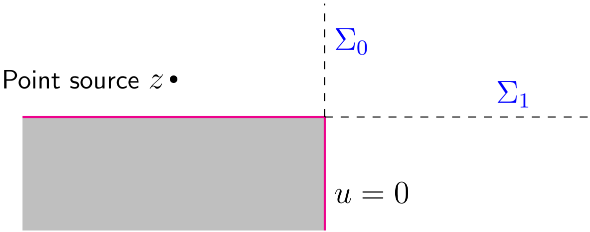
Let $\varphi_0(s) := u((0,s))$ and $\varphi_1(s) := u((s,0))$, for $s \geq 0$. Then, explicitly the above equations are ...

The HSMM integral equations



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\varphi_0(s) &= \psi(s) + \frac{\mathrm{i}ks}{2}\int_0^\infty \frac{H_1^{(1)}(k\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}\varphi_1(t)\,\mathrm{d}t, \quad s \geq 0,\\
\varphi_1(s) &= \frac{\mathrm{i}ks}{2}\int_0^\infty \frac{H_1^{(1)}(k\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}\varphi_0(t)\,\mathrm{d}t, \quad s \geq 0,
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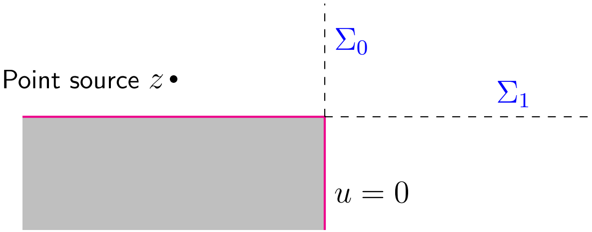

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with

$$\varphi_m(s) = a_m \mathrm{e}^{\mathrm{i}ks} s^{-1/2} + O(s^{-3/2}), \quad \text{as } s \to \infty, \quad m = 0, 1,$$

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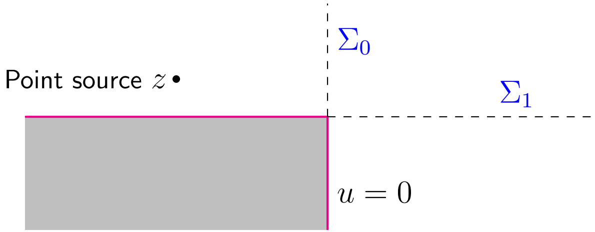

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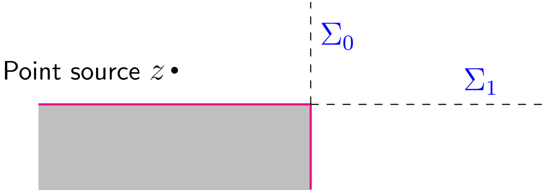
and

$$\psi(s) := \frac{\mathrm{i}}{4}H_0^{(1)}\left(k\sqrt{(s-z_2)^2+z_1^2}\right) - \frac{\mathrm{i}}{4}H_0^{(1)}\left(k\sqrt{(s+z_2)^2+z_1^2}\right), \quad s \geq 0.$$

The **Complex-Scaled** HSMM integral equations



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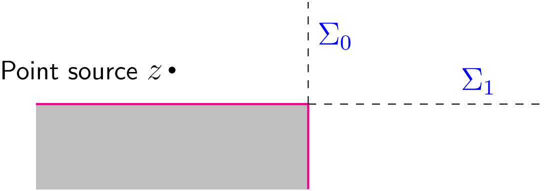

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1. Each RHS provides an **analytic continuation** of the LHS into the right-hand complex plane

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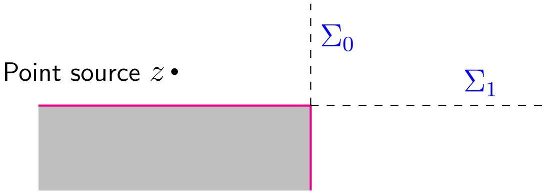

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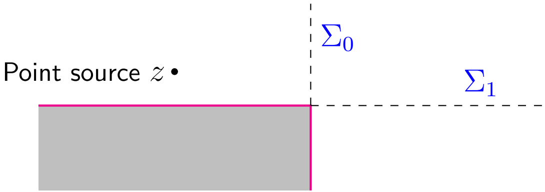

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2. Rotating the paths of integration we get ...

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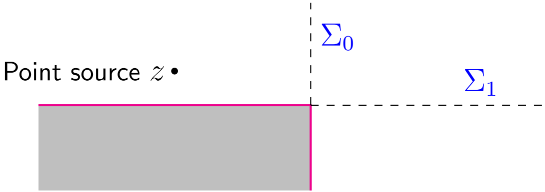

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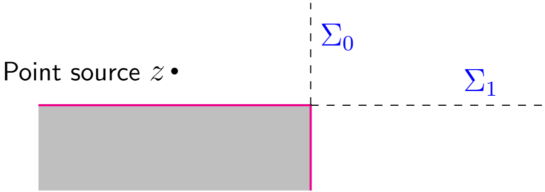
2. Rotating the paths of integration we get ...

The **Complex-Scaled** HSMM integral equations



$$\varphi_0(s) = \psi(s) + \frac{\mathrm{i}ks}{2}\int_0^{\mathrm{e}^{\mathrm{i}\theta}\infty} \frac{H_1^{(1)}(k\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}\varphi_1(t)\,\mathrm{d}t, \quad s = r\mathrm{e}^{\mathrm{i}\theta}, r \geq 0,$$

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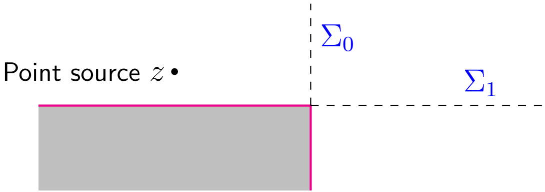
3. Introducing φ_m^θ and ψ^θ defined by $\varphi_m^\theta(r) := \varphi_m(r\mathrm{e}^{\mathrm{i}\theta})$ and $\psi^\theta(r) := \psi(r\mathrm{e}^{\mathrm{i}\theta})$, these equations are ...

The **Complex-Scaled** HSMM integral equations



$$\begin{aligned}\varphi_0^\theta(s) &= \psi^\theta(s) + \frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}\theta}s}{2}\int_0^\infty \frac{H_1^{(1)}(k\mathrm{e}^{\mathrm{i}\theta}\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}\varphi_1^\theta(t)\,\mathrm{d}t, \quad s \geq 0,\\ \varphi_1^\theta(s) &= \frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}\theta}s}{2}\int_0^\infty \frac{H_1^{(1)}(k\mathrm{e}^{\mathrm{i}\theta}\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}\varphi_0^\theta(t)\,\mathrm{d}t, \quad s \geq 0.\end{aligned}$$

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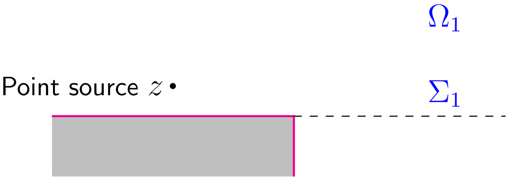
The **Complex-Scaled** HSMM integral equations



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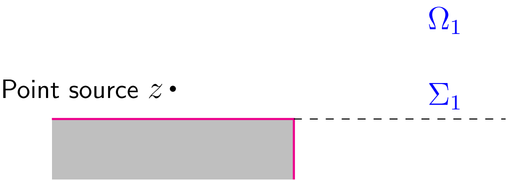
The **Complex-Scaled** HSMM integral equations



$$\varphi_0^{\theta}(s) \quad = \quad \psi^{\theta}(s) + \frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}\theta}s}{2}\int_0^{\infty}\frac{H_1^{(1)}(k\mathrm{e}^{\mathrm{i}\theta}\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}}\varphi_1^{\theta}(t)\,\mathrm{d}t, \quad s \geq 0,$$

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We can recover u: for example for $x \in \Omega_1$,

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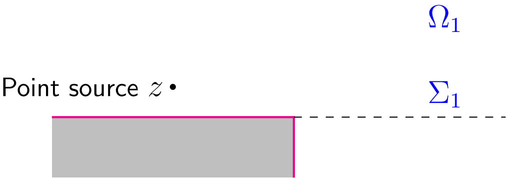

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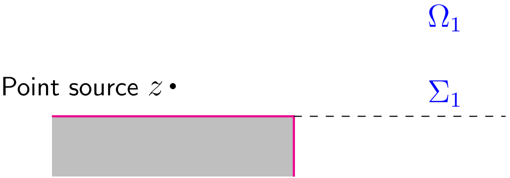

$$\varphi_0^{\theta}(s) = \psi^{\theta}(s) + \frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}\theta} s}{2} \int_0^\infty \frac{H_1^{(1)}(k\mathrm{e}^{\mathrm{i}\theta}\sqrt{s^2+t^2})}{\sqrt{s^2+t^2}} \varphi_1^{\theta}(t)\,\mathrm{d}t, \quad s \geq 0,$$

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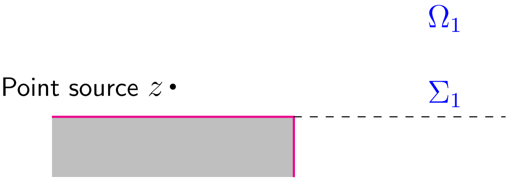

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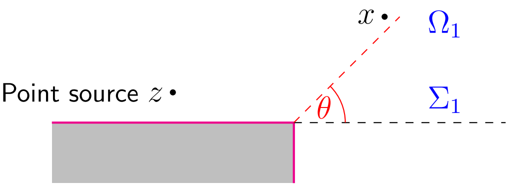

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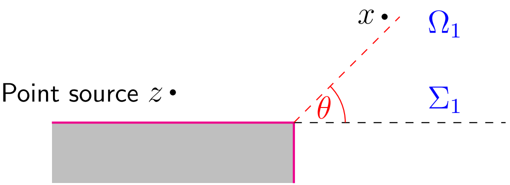
as long as $x_2 > \tan(\theta)\,x_1$.

The **Complex-Scaled** HSMM integral equations



$$
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\end{aligned}
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$$
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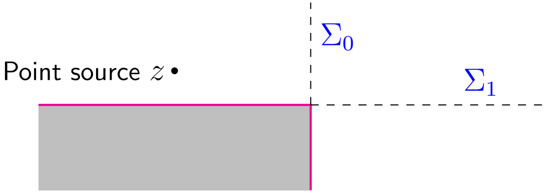
u(x) &= G(x,z) + 2\int_{\Sigma_1} \frac{\partial \Phi(x,y)}{\partial y_2} u(y)\,\mathrm{d}s(y)\\
&= G(x,z) + \frac{\mathrm{i}k\mathrm{e}^{\mathrm{i}\theta}x_2}{2}\int_0^\infty \frac{H^{(1)}(k\sqrt{x_2^2+(\mathrm{e}^{\mathrm{i}\theta}r-x_1)^2})}{\sqrt{x_2^2+(\mathrm{e}^{\mathrm{i}\theta}r-x_1)^2}}\varphi_1^{\theta}(r)\,\mathrm{d}r,
\end{aligned}
$$

as long as $x_2 > \tan(\theta)\,x_1$. **So take** $\theta < \pi/4$.

But why use the **CS** HSMM integral equations?



$$
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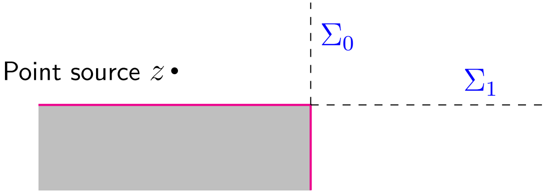

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Key feature. For some constant $C_\theta > 0$,

$$|\varphi_m^{\theta}(s)| \leq C_\theta \exp(-k\sin(\theta)), \quad s \geq 0, \quad m = 0, 1.$$

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Written in operator form these are

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$\varphi_0^\theta = \psi^\theta + D^\theta\varphi_1^\theta, \quad \varphi_1^\theta = D^\theta\varphi_0^\theta,$ i.e., $\begin{pmatrix} \varphi_0^\theta \\ \varphi_1^\theta \end{pmatrix} = \begin{pmatrix} \psi^\theta \\ 0 \end{pmatrix} + \mathbf{D}^\theta \begin{pmatrix} \varphi_0^\theta \\ \varphi_1^\theta \end{pmatrix}$

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Theorem. As an operator on $L^2(\mathbb{R}_+)$, $D^\theta = D_0 + D_1^\theta$ where D_1^θ is compact and

$$\|D_0\| = \frac{1}{\sqrt{2}}, \quad \|D_1^\theta\| \leq \frac{\sqrt{1-\mathrm{e}^{-\pi\sin(\theta)}}}{4\sqrt{\pi}\sin(\theta)},$$

so that $\|\mathbf{D}^\theta\| = \|D^\theta\| \leq \|D_0\| + \|D_1^\theta\| < 1$ if

$$\theta > \sin^{-1}(p/\pi) \approx 0.13438\pi,$$

where p is the unique positive solution of

$$\pi - \pi\mathrm{e}^{-p} = 8(3-2\sqrt{2})p^2.$$

But why use the **CS** HSMM integral equations?

The equations in operator form are

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As a consequence, if

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u can be recovered from φ_0^θ and φ_1^θ, and $\|\mathbf{D}^\theta\| < 1$ so Neumann iteration converges

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As a consequence, if

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u can be recovered from φ_0^θ and φ_1^θ, and $\|\mathbf{D}^\theta\| < 1$ so Neumann iteration converges, and Galerkin methods are convergent and quasi-optimal:

$$\text{Error in Galerkin solution} \leq \frac{\|\mathbf{D}^\theta\|}{1 - \|\mathbf{D}^\theta\|} \text{ Best approximation from Galerkin subspace}$$

The **CS** HSMM integral equations: numerical results

The equations in operator form are

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The **CS** HSMM integral equations: numerical results

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Approximate the integral operator D^θ by an $N \times N$ matrix D_N^θ by approximating

$$\int_0^\infty \approx \int_0^L \approx \text{Midpoint rule with } N \text{ subintervals}$$

and by collocating at the midpoints of the subintervals.

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The discrete unknowns are $N \times 1$ vectors $\boldsymbol{\varphi}_m^\theta$, $m = 0, 1$, approximations to the true values at the collocation points, that satisfy

$$\boldsymbol{\varphi}_0^\theta = \boldsymbol{\psi}^\theta + D_N^\theta \boldsymbol{\varphi}_1^\theta, \quad \boldsymbol{\varphi}_1^\theta = D_N^\theta \boldsymbol{\varphi}_0^\theta$$

The **CS** HSMM integral equations: numerical results

Approximate the integral operator D^{θ} by an $N \times N$ matrix D^{θ}_{N} by approximating

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The CS HSMM integral equations: numerical results

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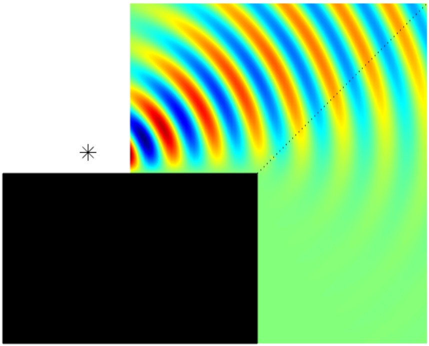
and by collocating at the midpoints of the subintervals.

Results for $L = 3$ wavelengths $= \frac{4\pi}{k}$, $\quad N = 20, \quad \theta = 0.24\pi.$



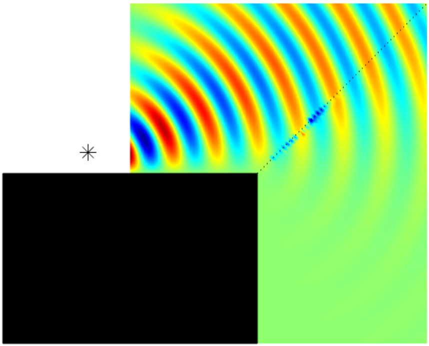
The **CS** HSMM integral equations: numerical results

Approximate the integral operator D^θ by an $N \times N$ matrix D^θ_N by approximating

$$\int_0^\infty \approx \int_0^L \approx \text{Midpoint rule with } N \text{ subintervals}$$

and by collocating at the midpoints of the subintervals.

Results for $L = 3$ wavelengths $= \dfrac{4\pi}{k}$, $\quad N = 20, \quad \theta = 0.27\pi.$



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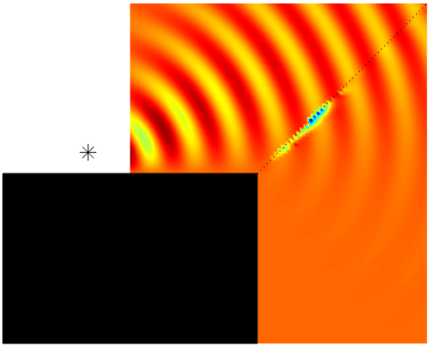

The CS HSMM integral equations: numerical results

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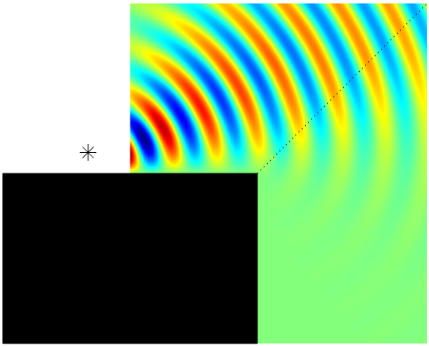
Results for $L = 3$ wavelengths $= \dfrac{4\pi}{k}, \quad N = 20, \quad \theta = 0.24\pi.$



What can the CS HSMM do apart from wedges?

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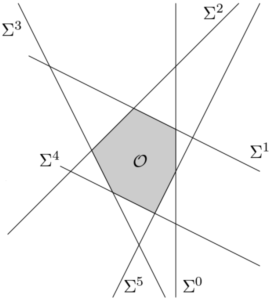
Polygons with Dirichlet (or other b.c.'s) in homogeneous medium



See Bonnet-Bendhia, C-W, Fliss, Hazard, Perfekt, Tjandrawidjaja, *SIAM J. Math. Anal.* 2022.

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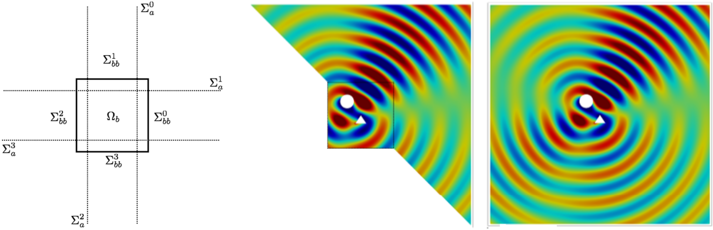
Arbitrary inhomogeneity in homogeneous medium



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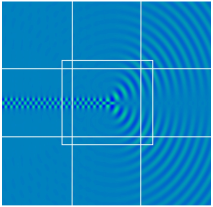

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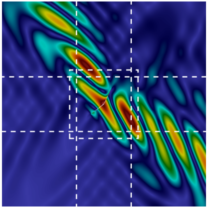

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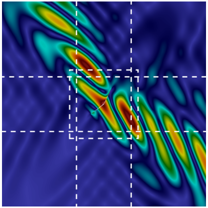

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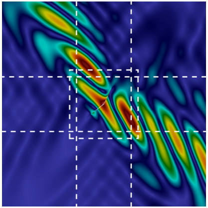

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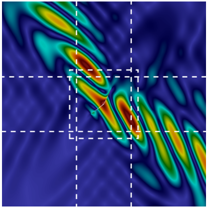

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So Happy 60th Éliane, Anne-Sophie, Christophe, Éric

Congratulations on ...

- Your excellent research over many years - e.g., the references above!
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And here's wishing you all the best, for a happy, successful, and productive future!