A contrast source inversion method for the reconstruction of electrical properties

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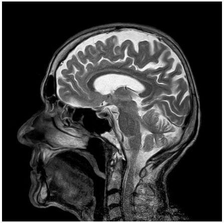
Outline

- Medical context
- The forward problem
- Contrast source inversion
- Conclusion

Medical context

Aim

Reconstruct the electrical properties (permittivity and conductivity) of the tissues in the human brain from radio-frequency measurements obtained by magnetic resonance imaging (MRI).



$\rightsquigarrow$ disease detection
$\rightsquigarrow$ safety standards

ANR Project ELECTRA : IADI (INSERM, Nancy), CHRU Nancy, Healtis (Nancy), ICUBE (Strasbourg), LMR (Reims)

numerical methods for precise reconstruction of electric properties & database with respect to age

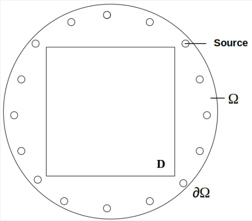
Magnetic resonance imaging (MRI)

Principle :

- strong static magnetic field $\mathbf{B}_0$, e.g. 3T
- excitation of hydrogen protons through radiofrequency (RF) pulse at Larmor frequency, e.g. 128 MHz at 3T
- emission of e.m. signal when protons return to initial state

$\Rightarrow$ picture of biological tissues containing water

Birdcage coil : a typical configuration of RF antenna



$\Omega \rightsquigarrow$ computational domain

$D \subset \Omega \rightsquigarrow$ measurement area

line source field from 8 or 16 legs (outside D)

Maxwell's equations

time-harmonic Maxwell's equations at fixed angular frequency ω with linear isotropic constitution laws, Ohm's law and scaling wrt electric permittivity ε_0 and magnetic permeability μ_0 in free space

$$\Rightarrow \qquad \mathcal{E}(\mathbf{x},t) = \Re e\left(e^{-i\omega t}\sqrt{\varepsilon_0}\mathbf{E}\right),\ \mathcal{H}(\mathbf{x},t) = \Re e\left(e^{-i\omega t}\sqrt{\mu_0}\mathbf{H}\right)$$

$$\begin{aligned} -ik\varepsilon_r \mathbf{E} - \operatorname{curl} \mathbf{H} &= -\sqrt{\mu_0}\mathbf{J}_s \\ -ik\mu_r \mathbf{H} + \operatorname{curl} \mathbf{E} &= 0 \end{aligned}$$

with source term $\mathbf{J}_s$, wave number $k = \omega\sqrt{\varepsilon_0\mu_0}$ and relative electromagnetic parameters

$$\varepsilon_r = \frac{1}{\varepsilon_0}\left(\varepsilon + \frac{i\sigma}{\omega}\right),\ \mu_r = \frac{\mu}{\mu_0}.$$

Elimination of H :

$$\operatorname{curl} \mu_r^{-1} \operatorname{curl} \mathbf{E} - k^2 \varepsilon_r \mathbf{E} = ik\sqrt{\mu_0}\mathbf{J}_s$$

Two-dimensional transverse magnetic setting and $\mu_r = 1$ **:**

$$\mathbf{E} = \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix}, \mathbf{J}_s = \begin{pmatrix} 0 \\ 0 \\ j_s \end{pmatrix} \Rightarrow -\Delta E - k^2 \varepsilon_r E = \underbrace{ik\sqrt{\mu_0}\, j_s}_{\stackrel{\text{def}}{=} F}$$

Perfect conductor boundary condition in a bounded domain Ω :

$$\begin{cases} -\Delta E - k^2 \varepsilon_r E &= F \quad \text{in } \Omega, \\ E &= 0 \quad \text{on } \partial\Omega. \end{cases}$$

Perturbation due to the presence of an object

Assume that $k^2 \notin \mathrm{sp}(-\Delta^{\mathrm{Dir}})$.

Reference configuration : no object $\Rightarrow \varepsilon_r = 1$. Source term F.

$$\left\{ \begin{array}{rcll} -\Delta E^{\mathrm{ref}} - k^2 E^{\mathrm{ref}} & = & F & \text{in } \Omega, \\ E^{\mathrm{ref}} & = & 0 & \text{on } \partial\Omega. \end{array} \right. \quad (1)$$

Configuration with object : assume that $\mathrm{supp}(1-\varepsilon_r) \subset D$ where $D \subset \Omega$. $\mathfrak{Im}\, \varepsilon_r > 0$ on some part of D. Same source term F.

$$\left\{ \begin{array}{rcll} -\Delta E^{\mathrm{tot}} - k^2 \varepsilon_r E^{\mathrm{tot}} & = & F & \text{in } \Omega, \\ E^{\mathrm{tot}} & = & 0 & \text{on } \partial\Omega. \end{array} \right. \quad (2)$$

Forward problem

Let E^{ref} be the solution to (1) for given F.
For **given** ε_r, find the **scattered field** $E^{\mathrm{sc}} = E^{\mathrm{tot}} - E^{\mathrm{ref}}$, solution to

$$\left\{ \begin{array}{rcll} -\Delta E^{\mathrm{sc}} - k^2 \varepsilon_r E^{\mathrm{sc}} & = & -k^2(1-\varepsilon_r) E^{\mathrm{ref}} & \text{in } \Omega, \\ E^{\mathrm{sc}} & = & 0 & \text{on } \partial\Omega. \end{array} \right. \quad (3)$$

Which formulation for the inverse problem ?

Inverse problem as a parameter problem

Let E^{ref} be the solution to (1) for given F.

For **given measurements** f^{data} on $D \subset \Omega$, find ε_r such that the scattered field E^{sc}, solution to (3), satifies $E^{\mathrm{sc}}_{|D} = f^{\mathrm{data}}$.

$\rightsquigarrow$ resolution by minimization of a least square functional of the data error.

Difficulty : inverse parameter problem, the differential operator in (3) depends on ε_r.

$$\left\{ \begin{array}{rcll} -\Delta E^{\mathrm{sc}} - k^2 \varepsilon_r E^{\mathrm{sc}} & = & -k^2(1-\varepsilon_r) E^{\mathrm{ref}} & \text{in } \Omega, \\ E^{\mathrm{sc}} & = & 0 & \text{on } \partial\Omega. \end{array} \right.$$

Reformulation of problem (3)

$$\left\{\begin{array}{rcll} -\Delta E^{\rm sc} - k^2 E^{\rm sc} & = & -k^2(1-\varepsilon_r)E^{\rm tot} & \text{in } \Omega, \\ E^{\rm sc} & = & 0 & \text{on } \partial\Omega. \end{array}\right. \quad (4)$$

Inverse problem as a non-linear source problem

Let $E^{\rm ref}$ be the solution to (1) for given F.

For **given measurements** $f^{\rm data}$ on $D \subset \Omega$, find $(\varepsilon_r, E^{\rm tot})$ defined on Ω, such that the scattered field $E^{\rm sc}$, solution to (4), satisfies the following system

$$\left\{\begin{array}{rcll} E^{\rm sc}_{|D} & = & f^{\rm data} & \textbf{[data equation]} \\ E^{\rm ref} + E^{\rm sc} & = & E^{\rm tot} & \textbf{[state equation]} \end{array}\right.$$

$\rightsquigarrow$ resolution by minimization of a least square functional of the weighted data and state error.

The inverse problem as contrast source inversion

Let $\chi = 1 - \varepsilon_r$ the contrast function and $w = \chi E^{\mathrm{tot}}$ the contrast source.

Problem (4) $\Rightarrow$ $$-\Delta E^{\mathrm{sc}} - k^2 E^{\mathrm{sc}} = -k^2 \underbrace{(1-\varepsilon_r)E^{\mathrm{tot}}}_{=\chi E^{\mathrm{tot}} \stackrel{\mathrm{def}}{=} w} \text{ in } \Omega.$$

Linear solution operator

$$\begin{array}{rcccc} \mathcal{L}_b & : & L^2(\Omega) & \to & H^1_0(\Omega) \\ & & w & \mapsto & E^{\mathrm{sc}} \end{array}$$

where $E^{\mathrm{sc}} \in H^1_0(\Omega)$ is the (variational) solution to

$$\left\{ \begin{array}{rcll} -\Delta E^{\mathrm{sc}} - k^2 E^{\mathrm{sc}} & = & -k^2 w & \text{in } \Omega, \\ E^{\mathrm{sc}} & = & 0 & \text{on } \partial\Omega. \end{array} \right. \quad (5)$$

The inverse problem as contrast source inversion

Let $\mathcal{R}_D : L^2(\Omega) \to L^2(D)$ be the restriction operator to $D \subset \Omega$.

Recall that $\chi = 1 - \varepsilon_r$ and $w = \chi E^{\mathrm{tot}}$.

Operator formulation of the data and state equation :

$$\left| \begin{array}{rcl} E^{\mathrm{sc}}_{|D} = f^{\mathrm{data}} & \Rightarrow & \mathcal{R}_D \circ \mathcal{L}_b(w) = f^{\mathrm{data}} \\ E^{\mathrm{ref}} + E^{\mathrm{sc}} = E^{\mathrm{tot}} & \Rightarrow & \chi\left(E^{\mathrm{ref}} + \mathcal{L}_b(w)\right) = w \end{array} \right.$$

Inverse problem as contrast source inversion (CSI)

Let E^{ref} be the solution to (1) for given F.

For **given measurements** f^{data} on $D \subset \Omega$, find (χ, w) defined on Ω, such that

$$\left\{ \begin{array}{rcll} \mathcal{R}_D \circ \mathcal{L}_b(w) & = & f^{\mathrm{data}} & \textbf{[data equation]} \\ \chi\left(E^{\mathrm{ref}} + \mathcal{L}_b(w)\right) & = & w & \textbf{[state equation]} \end{array} \right.$$

Contrast Source Inversion (CSI) - State of the art

Seminal paper of van den Berg/Kleinman, Inv. Pb., 1997

- reconstruction of the complex refraction index of an object from surface measurements outside the object

$\Rightarrow$ scalar scattering problem in an unbounded domain

- solution operator based on Green's function

Unbounded domain, MRI-EPT

- Balidemaj et al., IEEE Trans. Medical Imaging, 2016
- Arduino et al., Inv. Pb., 2018

Bounded domain - FEM

- Zakaria et al., Inv. Pb., 2010 (microwave imaging, Helmholtz equation)
- Arduino et al., IEEE Trans. Medical Imaging, 2021 (MRI-EPT)

Resolution of the inverse problem by minimization

Cost function

$$\mathcal{F}(\chi, w) = \mathcal{F}^{\text{data}}(w) + \mathcal{F}^{\text{state}}(\chi, w) \longrightarrow \min$$

with
$$\begin{aligned} \mathcal{F}^{\text{data}}(w) &= \frac{\eta^{\text{data}}}{2} \| f^{\text{data}} - (\mathcal{R}_D \circ \mathcal{L}_b)(w) \|_{0,D}^2, \\ \mathcal{F}^{\text{state}}(\chi, w) &= \frac{\eta^{\text{state}}}{2} \| \chi(E^{\text{ref}} + \mathcal{L}_b(w)) - w \|_{0,D}^2, \end{aligned}$$

for any $\chi \in L^\infty(\Omega)$ and $w \in L^2(\Omega)$ and normalization constants $\eta^{\text{data}} > 0$ and $\eta^{\text{state}} > 0$.

Two-step iterative method :

- update w_n by a gradient-based method $\rightsquigarrow w_{n+1}$
- update χ_n from knowledge of w_{n+1}

Computation of the gradient by the adjoint state

Adjoint solution operator

$$\begin{array}{rcccl} \mathcal{L}_b^* & : & H^{-1}(\Omega) & \to & L^2(\Omega) \\ & & \varphi & \mapsto & -k^2 p, \end{array}$$

where the adjoint state $p \in H_0^1(\Omega)$ is the variational solution of

$$\left\{ \begin{array}{rcll} -\Delta p - k^2 p & = & \varphi & \text{in } \Omega, \\ p & = & 0 & \text{on } \partial\Omega. \end{array} \right. \tag{6}$$

Then $(\mathcal{R}_D \circ \mathcal{L}_b)^*$ is defined with the help of the extension operator $\mathcal{R}_D^* : L^2(D) \to L^2(\Omega)$ given by

$$\mathcal{R}_D^*(\varphi) = \left\{ \begin{array}{ll} \varphi & \text{on } D \\ 0 & \text{on } \Omega \setminus \overline{D}. \end{array} \right.$$

Computation of the gradient by the adjoint state

$$\mathcal{F}^{\rm data}(w) = \frac{\eta^{\rm data}}{2}\|f^{\rm data} - (\mathcal{R}_D \circ \mathcal{L}_b)(w)\|_{0,D}^2,$$
$$\mathcal{F}^{\rm state}(\chi, w) = \frac{\eta^{\rm state}}{2}\|\chi(E^{\rm ref} + \mathcal{L}_b(w)) - w\|_{0,D}^2,$$

Formula for the gradient wrt w

$$\nabla_w \mathcal{F}^{\rm data}(w) = -\eta^{\rm data}\,\mathfrak{Re}\left\{(\mathcal{R}_D \circ \mathcal{L}_b)^*(\rho)\right\}$$
$$\nabla_w \mathcal{F}^{\rm state}(\chi, w) = -\eta^{\rm state}\,\mathfrak{Re}\left\{r - \mathcal{L}_b^*(\overline{\chi} r)\right\}$$

where $\rho = f^{\rm data} - (\mathcal{R}_D \circ \mathcal{L}_b)(w)$ is the data error and $r = \chi(E^{\rm ref} + \mathcal{L}_b(w)) - w$ is the state error which is taken into account only on the subdomain D.

Computation of the contrast function from given w

For a given contrast source w, the total field E^{tot} can be computed by

$$E^{\mathrm{tot}} = E^{\mathrm{ref}} + \mathcal{L}_b(w).$$

Then the contrast function

$$\chi = \frac{w}{E^{\mathrm{tot}}} = \frac{w\overline{E^{\mathrm{tot}}}}{|E^{\mathrm{tot}}|^2} \tag{7}$$

satifies the state equation

$$\chi\left(E^{\mathrm{ref}} + \mathcal{L}_b(w)\right) = w.$$

NB : For χ defined from w and E^{tot} by (7), the state error $r = \chi(E^{\mathrm{ref}} + \mathcal{L}_b(w)) - w$ vanishes. Thus,

$$\nabla_w \mathcal{F}^{\mathrm{state}}(\chi, w) = -\eta^{\mathrm{state}}\, \mathfrak{Re}\left\{r - \mathcal{L}_b^*(\overline{\chi} r)\right\} = 0.$$

CSI Algorithm

Data : E^{ref}, k, f^{data}. **Initial data :** w_0, χ_0

Initialization : compute $E_0^{\mathrm{sc}} = \mathcal{L}_b(w_0)$ and initial direction $v_0 = -g_0$

1. **Update :**
   - compute $\mathcal{L}_b(v_n)$
   - compute optimal step α_n (explicit formula depending on $\mathcal{L}_b(v_n)$)
   - update contrast source : $w_{n+1} = w_n + \alpha_n v_n$
   - solve direct problem : $E_{n+1}^{\mathrm{sc}} = E_n^{\mathrm{sc}} + \alpha_n \mathcal{L}_b(v_n)$
   - compute total field : $E_{n+1}^{\mathrm{tot}} = E^{\mathrm{ref}} + E_{n+1}^{\mathrm{sc}}$
   - compute new contrast function χ_{n+1} from formula (7)
2. **Compute new residual and gradient :**
   - $\rho_{n+1} = f^{\mathrm{data}} - E_{n+1|D}^{\mathrm{sc}}$
   - $g_{n+1} = -\eta^{\mathrm{data}} \mathfrak{Re} \left\{ (\mathcal{R}_D \circ \mathcal{L}_b)^* [\rho_{n+1}] \right\}$
3. **Non-linear conjugate gradient (Polak-Ribière) direction :**
   - $v_{n+1} = -g_{n+1} + \dfrac{< g_{n+1}, g_{n+1} - g_n >_D}{< g_n, g_n >_D} v_n$

First numerical results for synthetic electric field data

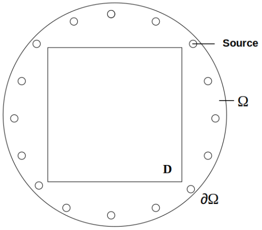


Figure – Configuration of a birdcage coil

Line source field

$$j_s(\mathbf{x}) = \sqrt{I} \sum_{\ell=1}^{16} \frac{e^{ikr_\ell}}{\sqrt{r_\ell}}$$

with intensity I and $r_\ell = \|\mathbf{x} - \mathbf{c}_\ell\|$ the distance to the center $\mathbf{c}_\ell$ of the ℓth leg.

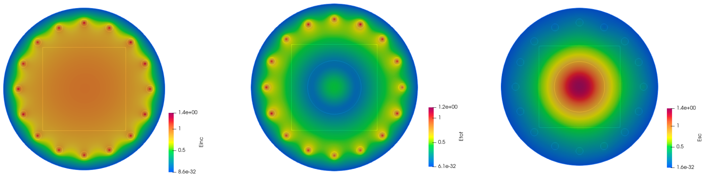


Figure – E^{ref} (without object, right), E^{tot} (with object, middle), E^{sc} (scattered field, right), P1 FEM - FreeFem++

Reconstruction of a homogeneous circular object with relative permittivity $\varepsilon_r = 52.5 + 48.028i$ (white matter at 128 [MHz]).

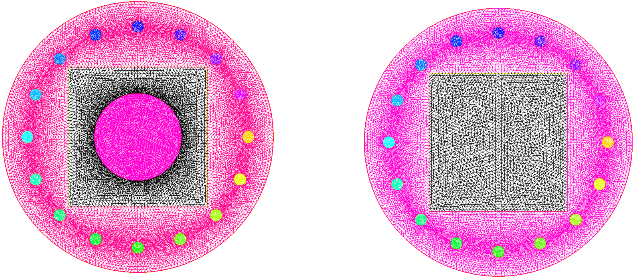


Figure – Mesh for data generation (left). Mesh for CSI algorithm (right), $\approx$ 2000 measurement points.

Reconstructed mean value of ε_r in the object : $51.98 + 47.26i$
$\rightsquigarrow \approx 1\%$ relative error for noiseless data

Noisy data

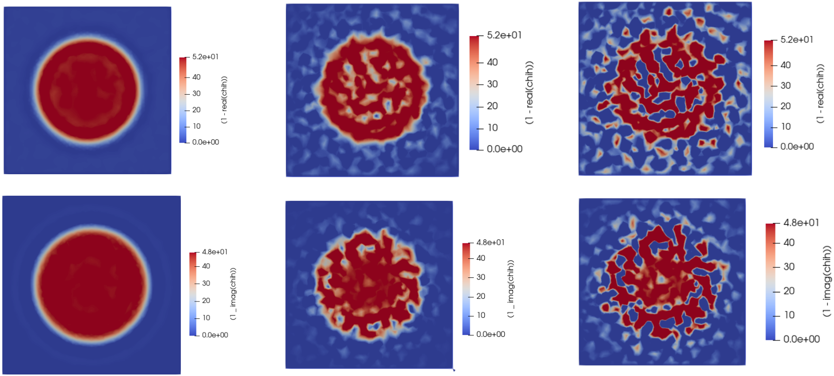


Figure – Noiseless data (left column). Noisy data with 0.5% (middle), 2% (right column). Upper line : real part of ε_r. Lower line : imaginary part of ε_r.

Realistic 2D head model

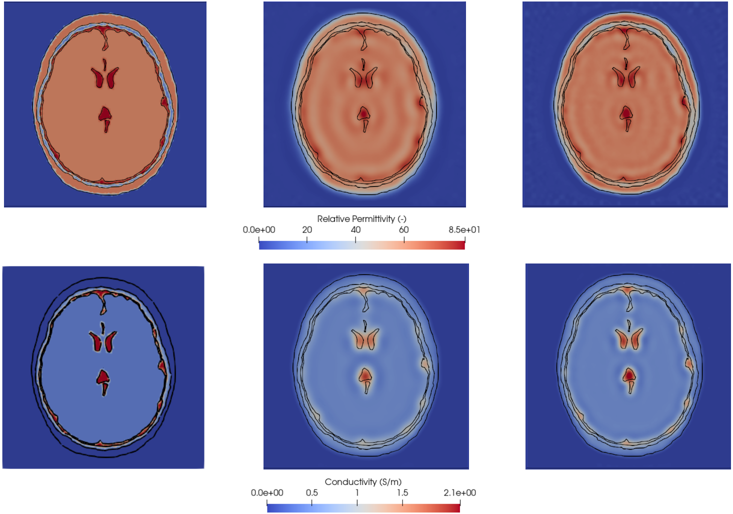


Figure – Permittivity (upper line) and conductivity (lower line). Noiseless data. Ground truth (left). 5000 iter. (middle). 20 000 iter. (right).

Realistic measurements

What does MRI really measure ?

MRI does not measure the electric field, but the B_1^+-field related to the scattered magnetic field.

Maxwell's equations ($\mu_r = 1$) $\quad \rightsquigarrow \quad \operatorname{curl} \mathbf{E} = ik\mathbf{H}$

$$\text{in 2D : } \mathbf{H} = -\frac{i}{k} \mathbf{curl}\, E = -\frac{i}{k} \begin{pmatrix} \partial_y E \\ -\partial_x E \end{pmatrix}.$$

MRI measurements

$$B_1^+ \stackrel{\text{def}}{=} \frac{H_x^{\text{sc}} + iH_y^{\text{sc}}}{2} \text{ on } D$$

Operator formulation of the data error with B_1^+-data

Define linear operators $\mathcal{C}$ and $\mathcal{P}$ by

Curl operator $\mathcal{C}$

$$\mathcal{C} : H_0^1(\Omega) \to L^2(\Omega)^2$$
$$E^{\rm sc} \mapsto -\frac{i}{k}\,\mathbf{curl}\, E^{\rm sc}.$$

B_1^+-operator $\mathcal{P}$

$$\mathcal{P} : L^2(\Omega)^2 \to L^2(\Omega)$$
$$\mathbf{u} \mapsto \frac{u_x + i u_y}{2}.$$

Then, the data operator $\mathcal{S}^+$ is defined from composition by

$$\mathcal{S}^+ : L^2(\Omega) \to L^2(D)$$
$$w \mapsto (\mathcal{R}_D \circ \mathcal{P} \circ \mathcal{C} \circ \mathcal{L}_b)(w)$$

Adjoint B_1^+-operator $\mathcal{P}^*$

$$\mathcal{P}^* : L^2(\Omega) \to L^2(\Omega)^2$$
$$\varphi \mapsto \frac{1}{2}\begin{pmatrix} \varphi \\ -i\varphi \end{pmatrix}.$$

Adjoint curl operator $\mathcal{C}^*$

$$\mathcal{C}^* : L^2(\Omega)^2 \to H^{-1}(\Omega)$$
$$\mathbf{v} \mapsto +\frac{i}{k}\,\mathrm{curl}\,\mathbf{v}.$$

with $\mathrm{curl}\,\mathbf{v} = \partial_x v_y - \partial_y v_x$.

The adjoint data operator

Data operator

$$\begin{aligned} \mathcal{S}^+ : L^2(\Omega) &\to L^2(D) \\ w &\mapsto (\mathcal{R}_D \circ \mathcal{P} \circ \mathcal{C} \circ \mathcal{L}_b)(w) \end{aligned}$$

$$\mathcal{S}^{+,*} = (\mathcal{L}_b^* \circ \mathcal{C}^* \circ \mathcal{P}^* \circ \mathcal{R}_D^*) : L^2(D) \to L^2(\Omega)$$

Let $\varphi \in L^2(D)$. Then, $\mathcal{S}^{+,*}(\varphi) = -k^2 p$ where p is the (variational) solution of

$$\left\{ \begin{array}{rcll} -\Delta p - k^2 p &=& \frac{i}{k} \operatorname{curl} \mathbf{v} & \text{in } \Omega, \\ p &=& 0 & \text{on } \partial\Omega. \end{array} \right. \tag{8}$$

where

$$\mathbf{v} = \mathcal{P}^*(\mathcal{R}_D^*(\varphi)) = \left\{ \begin{array}{ll} \frac{1}{2} \begin{pmatrix} \varphi \\ -i\varphi \end{pmatrix} & \text{on } D, \\ \mathbf{0} & \text{on } \Omega \setminus \overline{D}. \end{array} \right.$$

Cost function for B_1^+-data

$$\mathcal{F}(\chi, w) = \mathcal{F}^{\text{data},+}(w) + \mathcal{F}^{\text{state}}(\chi, w) \longrightarrow \min$$

with
$$\begin{aligned} \mathcal{F}^{\text{data},+}(w) &= \frac{\eta^{\text{data}}}{2} \| B_1^{+,\text{data}} - \mathcal{S}^+(w) \|_{0,D}^2, \\ \mathcal{F}^{\text{state}}(\chi, w) &= \frac{\eta^{\text{state}}}{2} \| \chi(E^{\text{ref}} + \mathcal{L}_b(w)) - w \|_{0,D}^2, \end{aligned}$$

Formula for the gradient wrt w

$$\begin{aligned} \nabla_w \mathcal{F}^{\text{data},+}(w) &= -\eta^{\text{data}} \, \Re e \left\{ \mathcal{S}^{+,*}(\rho^+) \right\} \\ \nabla_w \mathcal{F}^{\text{state}}(\chi, w) &= -\eta^{\text{state}} \, \Re e \left\{ r - \mathcal{L}_b^*(\overline{\chi} r) \right\} \end{aligned}$$

where $\rho^+ = B_1^{+,\text{data}} - \mathcal{S}^+(w)$ is the data error for B_1^+-data and $r = \chi(E^{\text{ref}} + \mathcal{L}_b(w)) - w$ is the state error (unchanged).

Finite Element Solver

FreeFem

F. Hecht, J. Num. Math., 2012.

Choice of Finite Elements ?

- Solution operator $\mathcal{L}_b$ and its adjoint $\mathcal{L}_b^*$:

$\rightsquigarrow$ discretize E^{sc} and p by Lagrange Finite Elements of type P1

- Restriction/extension operators $\mathcal{R}_D$ and $\mathcal{R}_D^*$:

$\rightsquigarrow$ implement the subdomain $D \subset \Omega$ as a region

- Implementation of the adjoint curl operator $\mathcal{C}^*$:

  A. Arduino et al., IEEE Trans. Medical Imaging, 2017.

  use Stokes' Theorem on a dual mesh and discretize $\mathcal{C}^*\mathbf{v}$ by Lagrange Finite Elements of type P1 with nodal values

$$(\mathcal{C}^*\mathbf{v})_I \stackrel{\text{def}}{=} \int_{G_I} \mathcal{C}^*\mathbf{v}\, dx = \frac{i}{k} \oint_{\partial G_I} \mathbf{v} \cdot \tau \, ds$$

Implementation of the adjoint curl operator

$\mathcal{C}^*$ is involved in the r.h.s. of the ajoint problem :

$$\left\{\begin{array}{rcll} -\Delta p - k^2 p & = & \frac{i}{k}\,\mathrm{curl}\,\mathbf{v} & \text{in } \Omega, \\ p & = & 0 & \text{on } \partial\Omega. \end{array}\right.$$

Variational formulation : Find $p \in H_0^1(\Omega)$ s.t.

$$a(q,p) = < q, \frac{i}{k}\,\mathrm{curl}\,\mathbf{v} >_{H_0^1(\Omega), H^{-1}(\Omega)} \quad \forall q \in H_0^1(\Omega),$$

where

$$a(q,p) = \int_\Omega \nabla q \cdot \nabla \overline{p}\, dx - k^2 \int_\Omega q \overline{p}\, dx \;\forall p, q \in H_0^1(\Omega).$$

$$< q, \frac{i}{k}\,\mathrm{curl}\,\mathbf{v} >_{H_0^1(\Omega), H^{-1}(\Omega)} = -\frac{i}{k} \int_\Omega \mathbf{curl}\, q \cdot \overline{\mathbf{v}}\, dx$$

$\rightsquigarrow$ discretize p and q by Lagrange Finite Elements of type P1

Numerical results for B_1^+-data

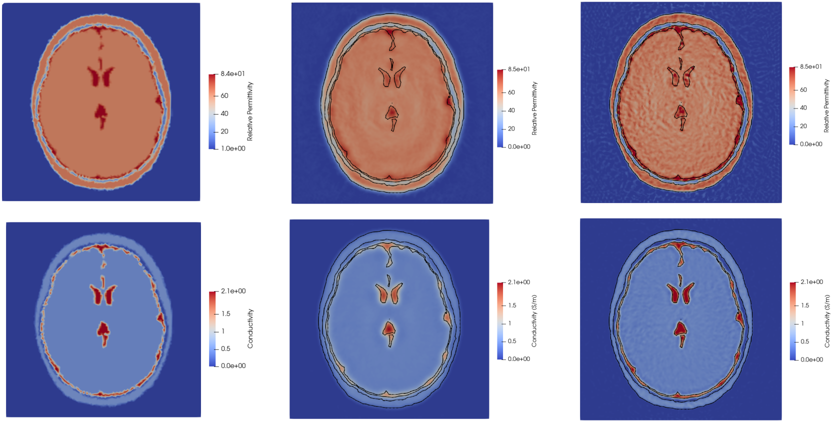
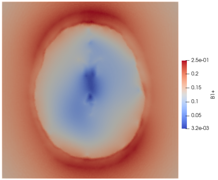


Table – Permittivity (upper line). Conductivity (lower line). Ground truth (left). Reconstruction after 40 (middle) and 100 (right) iterations. Refined mesh, $\approx$ 27 000 measurement points.

- Reconstruction of EPs ε and σ in various academic and realistic configurations.
- Good performance for noiseless data and electric measurements.
- First step towards realistic B_1^+-data.



$|B_1^+|$

In (near) future

- (Multiplicative) regularization of CSI [cf. Balidemaj et al 2016]
- Phaseless data : $|B_1^+|$ instead of full B_1^+ data [cf. Arduino et al. 2018]
- Experimental data of a phantom [IADI, Nancy]


Happy
Birthday